A Calculation of the Force Required to Raise the USTower HDX572 Amateur Radio Tower from the Horizontal Position as a function of the Tower Angle.

Revision 4.0

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DISCLAIMER: I am not a licensed nor registered Professional Engineer. I am not even an engineer. These calculations were an exercise in physics to satisfy my own personal curiosity. DO NOT USE THESE CALCULATIONS AS A BASIS FOR ANY DECISIONS OR ACTIONS REGARDING A TOWER INSTALLATION OR OPERATION!!!

Table of Contents

Forces Associated with Raising a Tiltover Tower

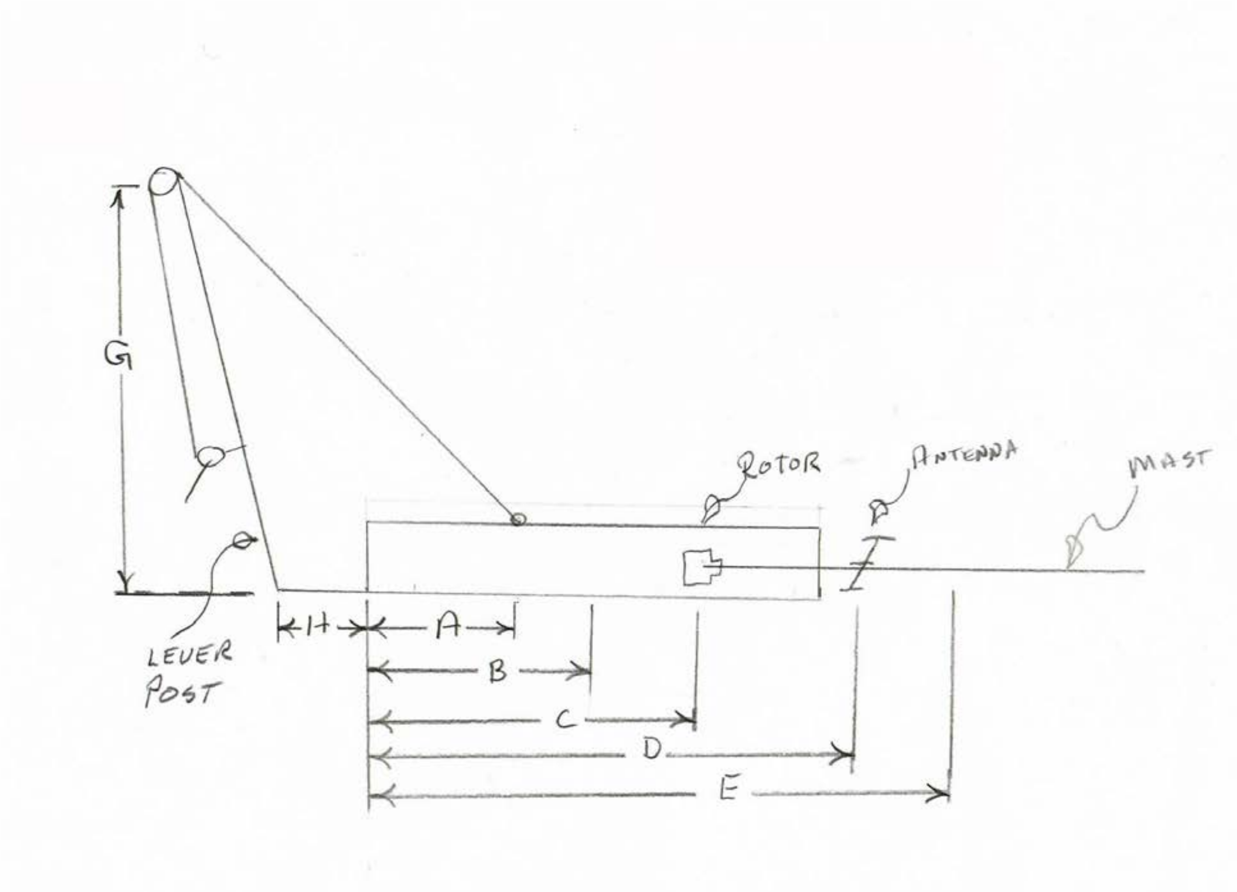
US Tower Model HDX-572 MDPL

Version 4

1. The Problem:

When the tower in Figure 1 is tilted over for maintenance, a hand crank hoist and pulley system is used to raise the tower back to its vertical position. It requires in excess of 400 revolutions of the hand crank to go from the horizontal to the vertical position. The majority of those revolutions require two hands to accomplish. It would be advantageous to replace the hand crank system with an electric hoist. Hoists are rated by their vertical lift capability. Therefore, in order to select the proper hoist, we need to know the maximum tension on the cable.

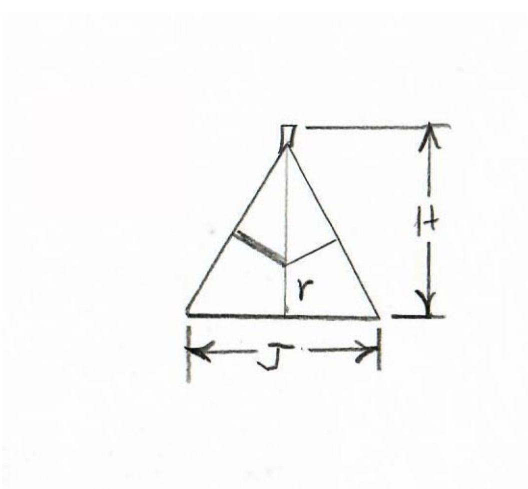
Figure 1. Tower Layout in the Horizontal Position.



Notes:

1. WARNING!! A winch is not a hoist and should never be used for lifting. A hoist is made and rated for vertical lifts. A winch is made and rated for horizontal pulls. The braking system on a hoist is, by design, more secure and reliable. The ratings on winches are somewhat arbitrary and are not standardized. A winch may be rated for 9000 lbs but there is no way it could lift 9000 lbs. It is rated for how much it could pull, BUT there is no specification regarding the friction against which it is pulling. Pulling 9000 lbs on a horizontal, frictionless surface is child's play as compared to typical soil conditions.
2. The tower layout is shown in Figure 1.
3. Tower loads include the tower, a rotor, the antenna and a mast.
4. The center of mass (COM) for each load is described.
5. The COMs are located on a line represented by the centroid of the triangular cross section of the tower, which is shown in Figure 2. The centroid is where the perpendicular lines from the center of each side intersect with the tower.
6. There are four cables that run between the Lift Pulleys (sheaves) on the top of the Lever Post and the Tower Pulleys on the side of the tower at the Attachment Point. There is one cable running from the Lift Pulleys at the top of the Lever Post to the hand crank. This is results in a 4:1 mechanical advantage at the hand crank.
7. The torques and forces pulling the tower away from the hand crank are, in this analysis called clockwise torques and forces. The torques and forces pulling toward the hand crank are, in this analysis, called counterclockwise torques and forces to correlate with the figures as drawn.

Figure 2, Tower Cross Section Showing the Centroid



2. Table of Symbols and Variables

A = Horizontal distance from the pivot to the actual tower cable Attachment Point.

A_V = Horizontal distance between the pivot and the Virtual Attachment Point.

B = Horizontal distance from the pivot to the tower COM.

C = Horizontal distance from the pivot to the rotor COM.

D = Horizontal distance from the pivot to the antenna COM.

E = Horizontal distance from the pivot to the mast COM.

F_C = The counterclockwise for that is applied to the cable bundle between the Attachment Point and the Lift Pulleys

F_T = The counterclockwise force coincident with torque T at the Attachment Point.

G = Length of the Lever Post.

G_Y = Vertical Height of the Lift Pulleys above the Common Reference Point.

G_X = Horizontal displacement of the Lift Pulleys relative to the Common Reference Point

H = Horizontal distance from the pivot to the Lever Post.

J = Distance between tower legs.

Lx = X-axis distance between the Attachment Point and the Lift Pulley,

Ly = Y-axis distance between the Attachment Point and the Lift Pulley.

P = Horizontal distance between pivot and Lift Pulley.

r = Distance between the middle of the side of the tower to the centroid of the triangular cross section.

T = The net counterclockwise torque.

α = The angle between the tower and horizontal

β = The angle between the Lever Post and vertical. AKA, the Lever Post retreat angle.

θ = The angle between the tower and the cable bundle

ψ = The angle between the cable bundle and horizontal

$\angle Av$ = The angle between the pivot and the Virtual Attachment Point, A_V.

$\angle B$ = The angle between the pivot and the COM of the tower, along the centroid axis.

$\angle C$ = The angle between the pivot and the COM of the rotor, along the centroid axis.

$\angle D$ = The angle between the pivot and the COM of the antenna, along the centroid axis.

$\angle E$ = The angle between the pivot and the COM of the mast, along the centroid axis.

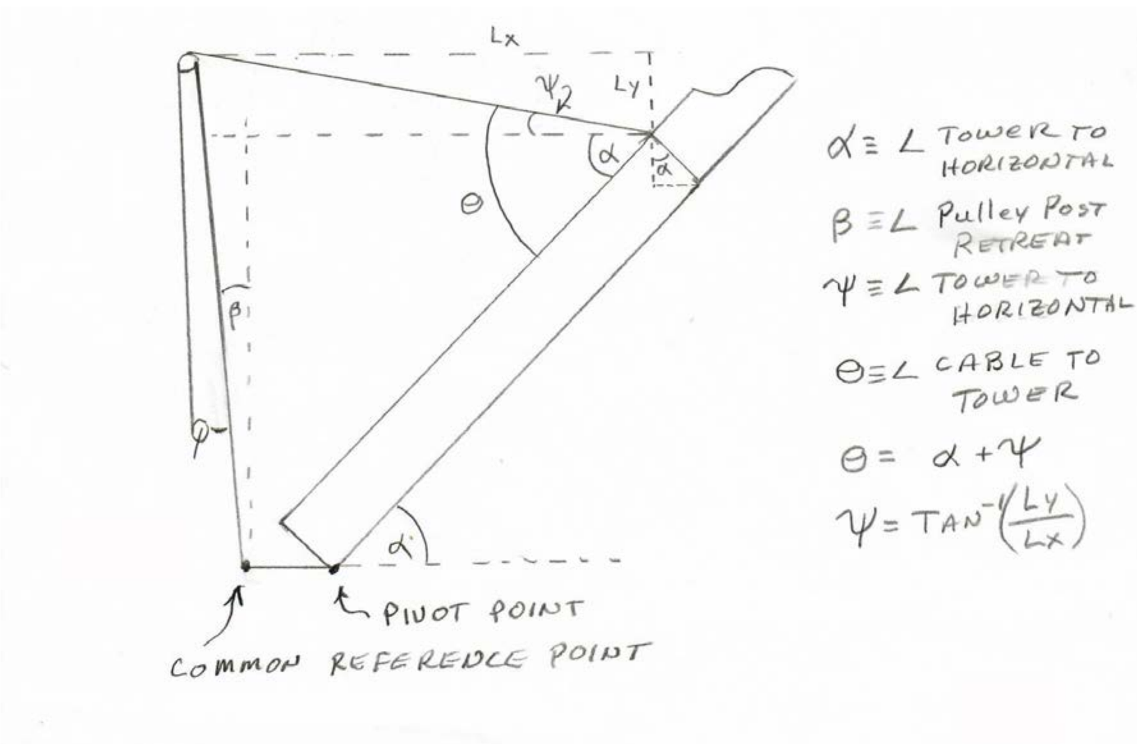
3. The Relationship Between α, θ and ψ

Definitions

$$\theta = \alpha + \psi$$

$$\psi = \text{Tan}^{-1}\,(Ly/Lx)$$

Figure 3. The relationship between the Angles α, θ and ψ



Alpha, α, is the angle between the tower and horizontal. See Figure 3. When the tower is vertical, α = 90°. θ is the angle between the tower and the cable bundle. The difference between these angles (θ – α) is the angle ψ. The angle ψ is also the angle between the cable bundle and horizontal.

In order to calculate ψ, we need to know the (x,y) positions of both the Lift Pulleys and the Actual Attachment Point, relative to a common reference point, which is defined as the base of the Lever Post. From the knowledge of these positions we can calculate the displacement of the Lift Pulleys from the Attachment Points in terms of the X-axis distance, Lx and the Y-axis distance, Ly.

Calculation of Ly

Ly is the vertical distance between the cable Attachment Point on the side of the tower to the Lift Pulleys on the top of the Lever Post.

This calculation starts with the height of the Lift Pulleys, Gy. The Lift Pulleys are on top of the Lever Post, with the common reference point being the base of the Lever Post. The Lever Post, with length G, is tilted at an angel, β, resulting in a reduction of the Lift Pulley height to:

$$Gy = G \bullet \text{Cos } \beta$$

The vertical distance of the Attachment Point relative to the common reference point is a result of the tower tilt angle, its horizontal distance from the Pivot ($A \bullet (\text{Sin } \alpha)$) , and its vertical displacement from the ground side of the tower ($H \bullet \text{Cos } \alpha$).

Attachment Point Displacement: $\Delta y = A \bullet (\text{Sin } \alpha) + H \bullet (\text{Cos } \alpha)$

Therefore,

$$Ly = Gy - (A \bullet (\text{Sin } \alpha) + H \bullet (\text{Cos } \alpha))$$

Calculation of Lx

Lx is the horizontal distance between the cable Attachment Point and the Lift Pulleys.

The shift if the Lift Pulley position away from the tower, as a result of the Lift Post retreat angle is:

$$Gx = G \bullet \text{Sin } \beta$$

The Attachment Point distance from the pivot in the X-Axis is given by:

$$\Delta x = H + A \bullet (\text{Cos } \alpha) - H \bullet (\text{Sin } \alpha)$$

Therefore:

$$Lx = Gx + \Delta x$$

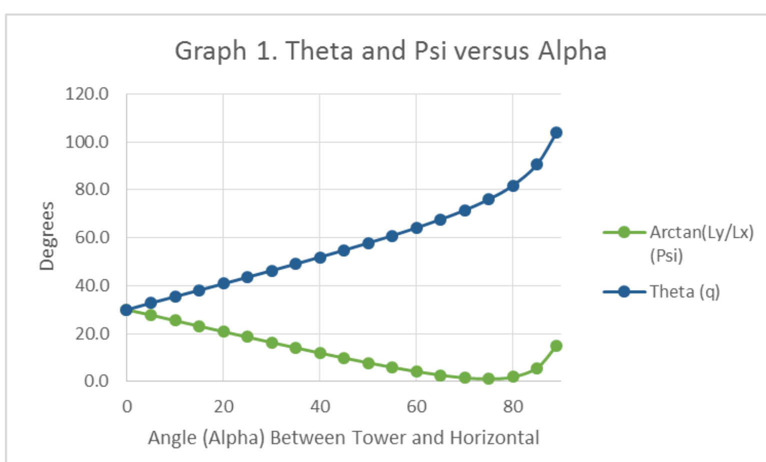
$$Lx = G \bullet \text{Sin } \beta + P + A \bullet (\text{Cos } \alpha) - H \bullet (\text{Sin } \alpha)$$

Where P is the distance between the Pivot and the Common Reference Point.

Calculation of the Angle ψ.

$$\psi = \text{Arctan } (Ly/Lx)$$

Graph 1 shows the size of Theta and Psi as a function of Alpha.



4. The Virtual Attachment Point

Assumption: The Counterclockwise torque, T, should be calculated from a point at the intersection of 1) a line through the center of the Lift Pulley and the Tower Attachment Pulley (Point) and 2) the Center of Mass (COM) axis which is coincident with the centroid of the triangular tower cross section.

The COMs are located on a line running down the length of the tower connecting the Centroid points of the triangular cross sections of the tower. The Centroid of a triangle is defined as the point where lines perpendicular to the center of the tower sides intersect inside the tower. The distance from the center of each side of the tower to the centroid point or line is given by r= side length times the square root of three, all divided by 6. See Figure 2.

r = (J x SQRT(3))/6

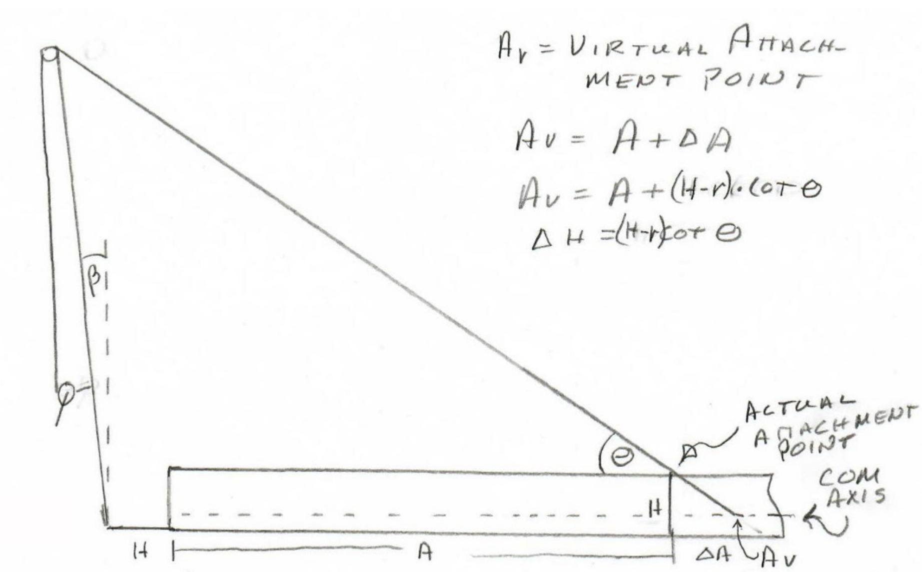
The location of the Virtual Attachment Point is a function of Theta, (Figure 4). The displacement distance ΔA along the COM axis line, from the actual attachment point, (distance A from the Pivot Point along the COM axis) for the Virtual Attachment Point is:

$\Delta A = (H-r) \bullet \cot\theta$

Therefore the COM axis distance from the Pivot Point to the Virtual Attachment Point is Av:

$Av = A + \Delta A = A + (H-r) \bullet \cot\theta$

Figure 4. The Virtual Attachment Point, Av.



5. Torque Analysis

For a stationary system the clockwise torque is equal to the counterclockwise torque.

$$\sum T_{\text{counterclockwise}} = \sum T_{\text{clockwise}}$$

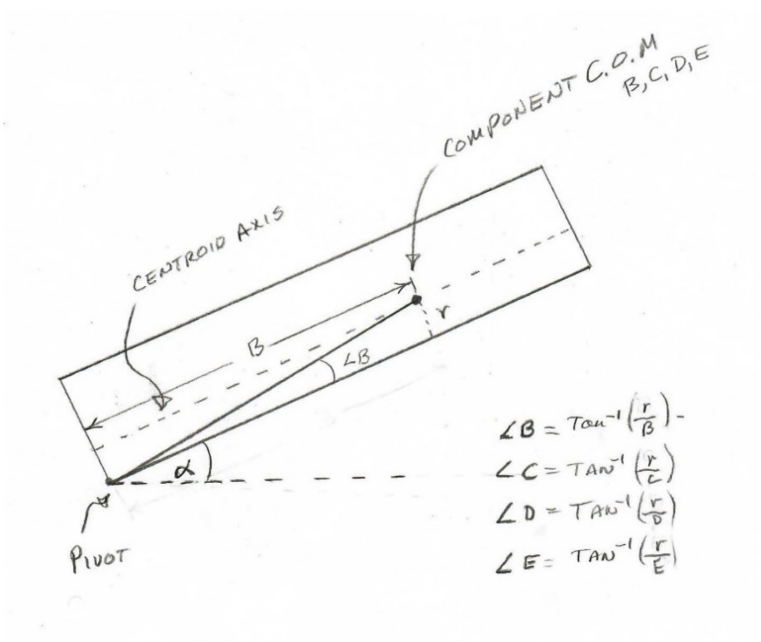
A. Clockwise Torque Calculations

The torque is calculated from the center of mass of the components of the system; the tower, the rotor, the antenna and the mast. The values of the torque are perpendicular to a line from the Pivot Point to the COM of the components. The force vectors used in these torque calculations are therefore also perpendicular to that same line.

In order to calculate that force vector we need to know the angle of the line from the Pivot Point to the COM of each component. See Figure 5. If the tower were horizontal we can calculate that angle since it is the angle from the side of the tower to the line from the Pivot Point to the COM. We will also need to know the angle for the Virtual Attachment Point, Av. For Each Component, the angle is given by the Arctan of (r/the x-axis distance to that components COM);

∠ Av = Tan-1 (r/Av)
∠ B = Tan-1 (r/B)
∠ C = Tan-1 (r/C)
∠ D = Tan-1 (r/D)
∠ E = Tan-1 (r/E)

Figure 5. The Component Angles for Torque Calculations



At any given tower angle, α, the total angle for the torque calculation will be, for example, For Av the angle to ground will be = (∠ Av + α) .

The total clockwise torque is the sum of the individual component torques:

$T = T_B + T_C + T_D + T_E$

Where T_B = Tower Torque,

T_C = Rotor Torque,

T_D = Antenna Torque, and

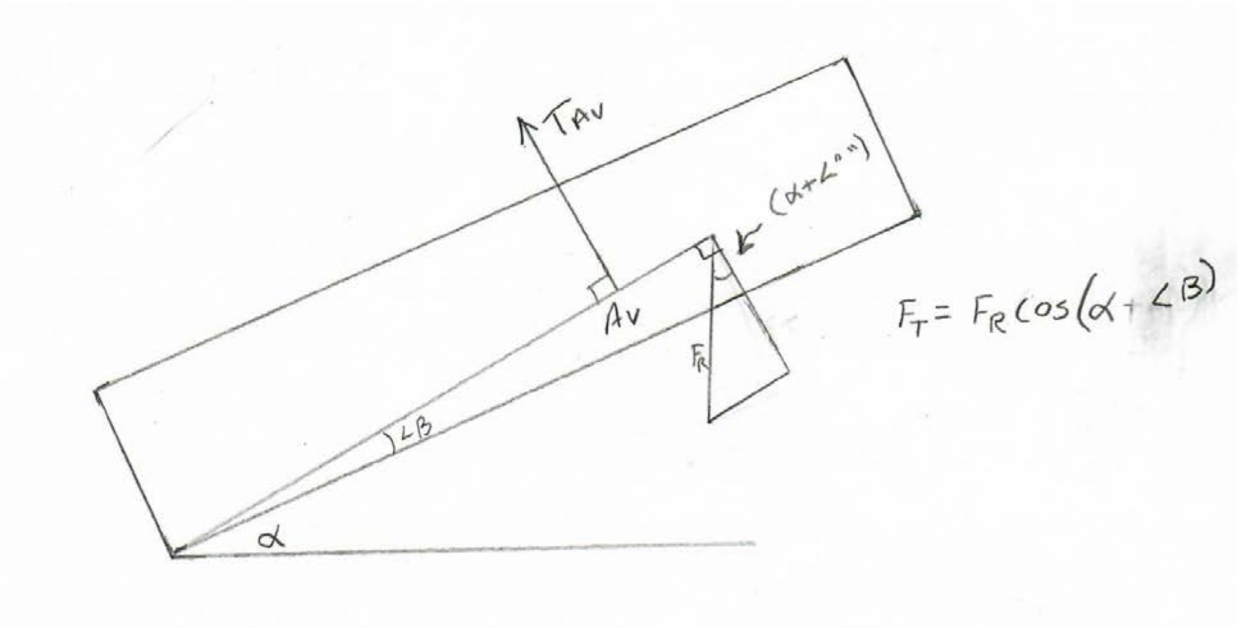
T_E = Mast Torque.

For any given component (e.g. The Tower, B) the torque will be the distance from the Pivot to the COM of the component times to force perpendicular to the line from the Pivot to the COM.

The distance will be the horizontal distance to the component center of mass, B, divided by the Cos (∠ B): See Figure 5

= B/ Cos (∠ B). This will be true for the Av as well.

Figure 6. The Individual Component Force Calculation



The force will be the weight of the component times the Cos (α + ∠ B). See Figure 6

Therefore the torque, T_B, will be:

T_B = (B/ Cos (∠ B)) • WT_B • Cos (α + ∠ B)

T_C = (C/ Cos (∠ C)) • WT_C • Cos (α + ∠ C)

T_D = (D/ Cos (∠ D)) • WT_D • Cos (α + ∠ D)

T_E = (E/ Cos (∠ E)) • WT_E • Cos (α + ∠ E)

B. Counterclockwise Torque and Force Calculations

The counterclockwise torque, T, is perpendicular to the line from the Pivot Point to the Virtual Attachment Point, Av, and will be equal to the force vector at the Virtual Attachment Point, F_T times the distance from the Pivot Point to the Virtual Attachment Point. F_T is parallel to T. See Figure 7

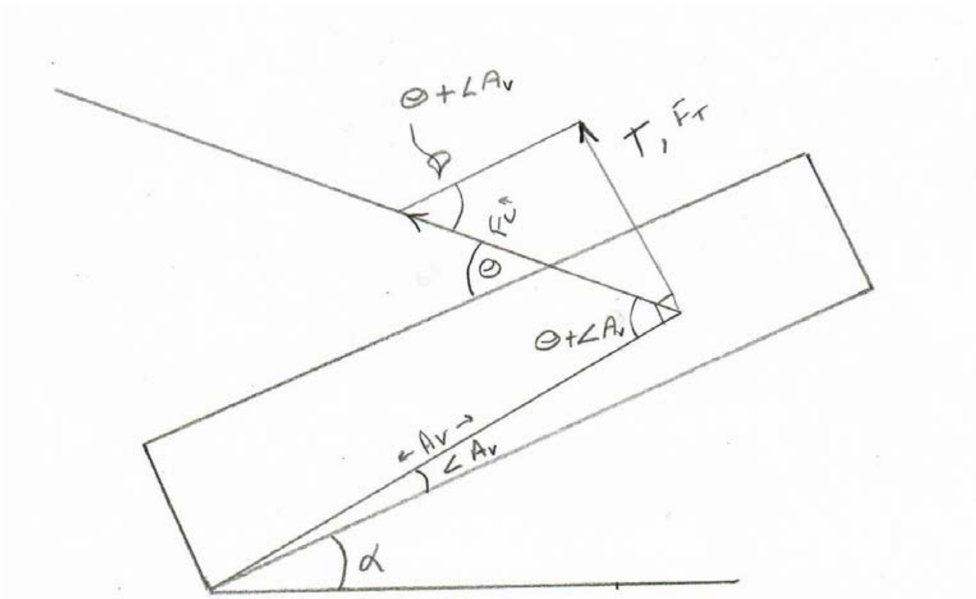
The distance from the Pivot Point to the Virtual Attachment Point is:

$= Av/\cos(\angle Av)$

$T = F_T \bullet Av/\cos(\angle Av) = \sum T_{clockwise}$

$F_T = \sum T_{clockwise} / (Av/\cos(\angle Av))$

Figure 7. Counterclockwise Force Calculation Diagram.



Fc is the tension on the cable bundle. To calculate Fc we need to know the angle between Fc and F_T.

The angle between the tower and the cable is θ. The angle between the tower and the line between the Pivot Point and the Virtual Attachment Point is ∠ Av. By inspection of Figure 7, it can be shown that the angle between Fc and F_T is $(90 - (\theta + \angle Av))$. The angle opposite that in the right triangle is $(\theta + \angle Av)$.

Therefore:

$F_T/Fc = \sin(\theta + \angle Av)$

Solving for force on the cable bundle, Fc gives:

$Fc = F_T/\sin(\theta + \angle Av)$

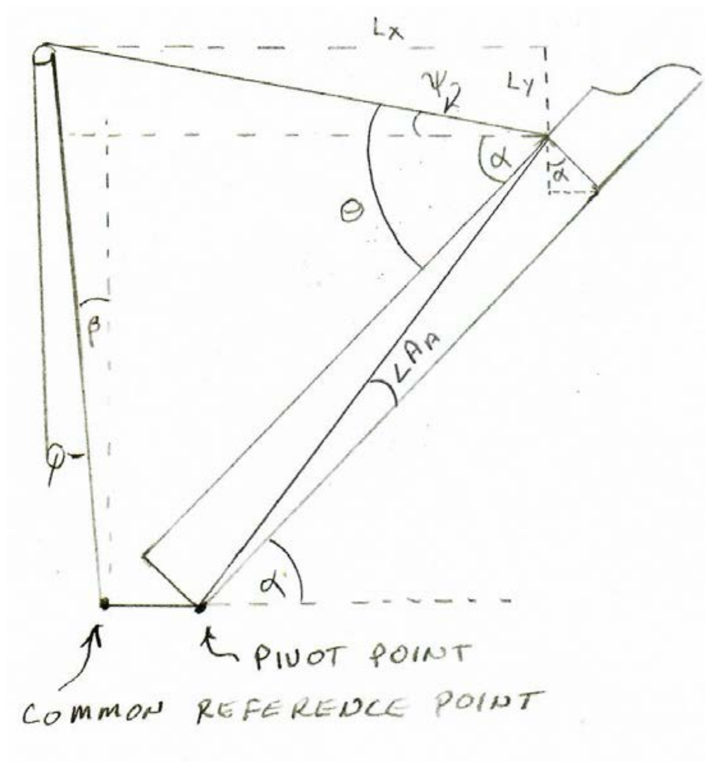
Finally, the force on the Hoist Cable is Fc/4.

6. Virtual vs Actual Attachment Points

In an earlier section an assumption was made that the attachment point used for the Counterclockwise Torque and Force calculations would need to be a virtual point along the line representing the center of mass of the various components. Indeed the analysis was done and the data shown in the Excel spread sheet, Chart 1, was generated using the virtual attachment point, which varied with the tilt angle alpha, α.

The insight into the problem as led to question whether this assumption was necessary, or whether the actual attachment point could be used for the calculations. In order to do this, 1) the Counterclockwise Torque would have to be perpendicular to a line from the Pivot Point to the Actual Attachment point (Line PA), 2) the lever arm length would have to be the actual distance from the Pivot Point to the Actual Attachment Point, Line A_A and 3) the angle between the Cable Bundle and Line PA would have to be known.

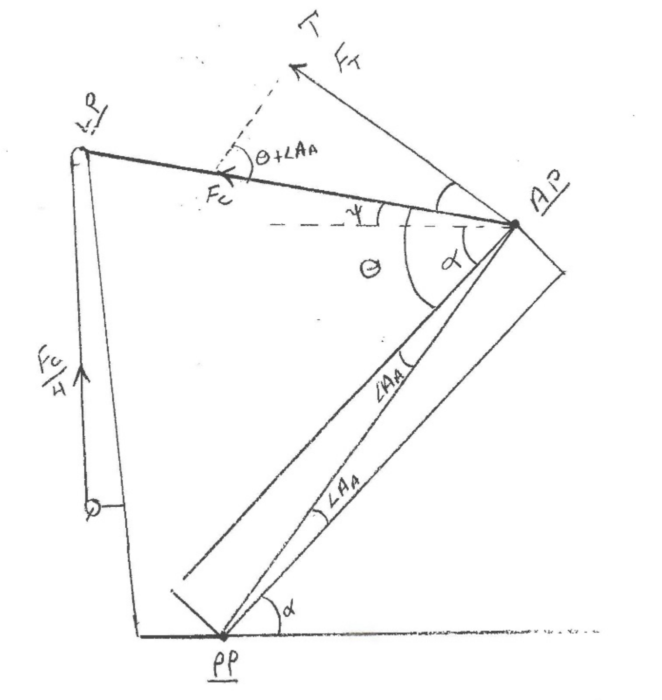
Figure 8. Angles associated with the Actual Attachment Point Calculations



Let's first consider the third item, the angle between the Line A_A and the Cable Bundle between the Actual Attachment Point and the Lift Pulley as shown in Figure 8. The angle between those two lines is the sum of two angles, θ and $\angle A_A$. The angle $\angle A_A$ can be calculated since we know length of the two sides of the triangle. The Adjacent Side is the distance A and the opposite side is the distance H. Therefore angle $\angle A_A$ = Arctan (H/A). For the current tower under consideration this is about 15.9°. The length of A_A is equal A/Cos($\angle A_A$) which is about 7.3 ft. The angle between horizontal and A_A is equal to $\alpha + \angle A_A$.

The first condition was that the Counterclockwise Torque would be perpendicular to the Line A_A. So the question becomes, what is the angle between the Cable Bundle and this Torque at the Actual Attachment Point.

Figure 9. Final Force Calculation Angles and Vectors from Actual Attachment Point.



As shown in Figure 9, the Line PP-AP, is perpendicular to the Torque line, Line AP-LP. Therefore, the angle between the Torque vector originating at point AP, in Figure 9 and the Cable Bundle, shown as Line AP-LP, is equal to 90° - ($\theta + \angle A_A$). Also, note that the complementary angle opposite this angle is just ($\theta + \angle A_A$).

So now we can calculate the forces involved.

The Counterclockwise Torque is equal to the Clockwise Torque and is equal to F_T times the lever arm distance, A_A.

$$T = F_T \bullet A_A$$

And

$$F_T = T/A_A$$

$$\sin(\theta + \angle A_A) = F_T / Fc$$

$$Fc = F_T / \sin(\theta + \angle A_A)$$

And the force on the winch cable is 1/4th the force on the Cable Bundle

$$Fw = Fc/4$$

7. Summary and Conclusions

An Excel model was prepared from these calculations which is shown in Chart 1. The results of the model calculations seem reasonable. The maximum force on the cable with the tower at 0°, or horizontal, is about 1650 lbs. As the tower approaches vertical the model predicts that this force will go negative. This is consistent with observations. When the tower reaches a certain angle the center of mast goes past the Pivot Point and it is necessary to use a hydrolic jack to keep the tower from slamming into position.

There has been a lot of learning by the author during this exercise to analyze this situation and get an answer. I think my understanding of torque problems has grown nicely.

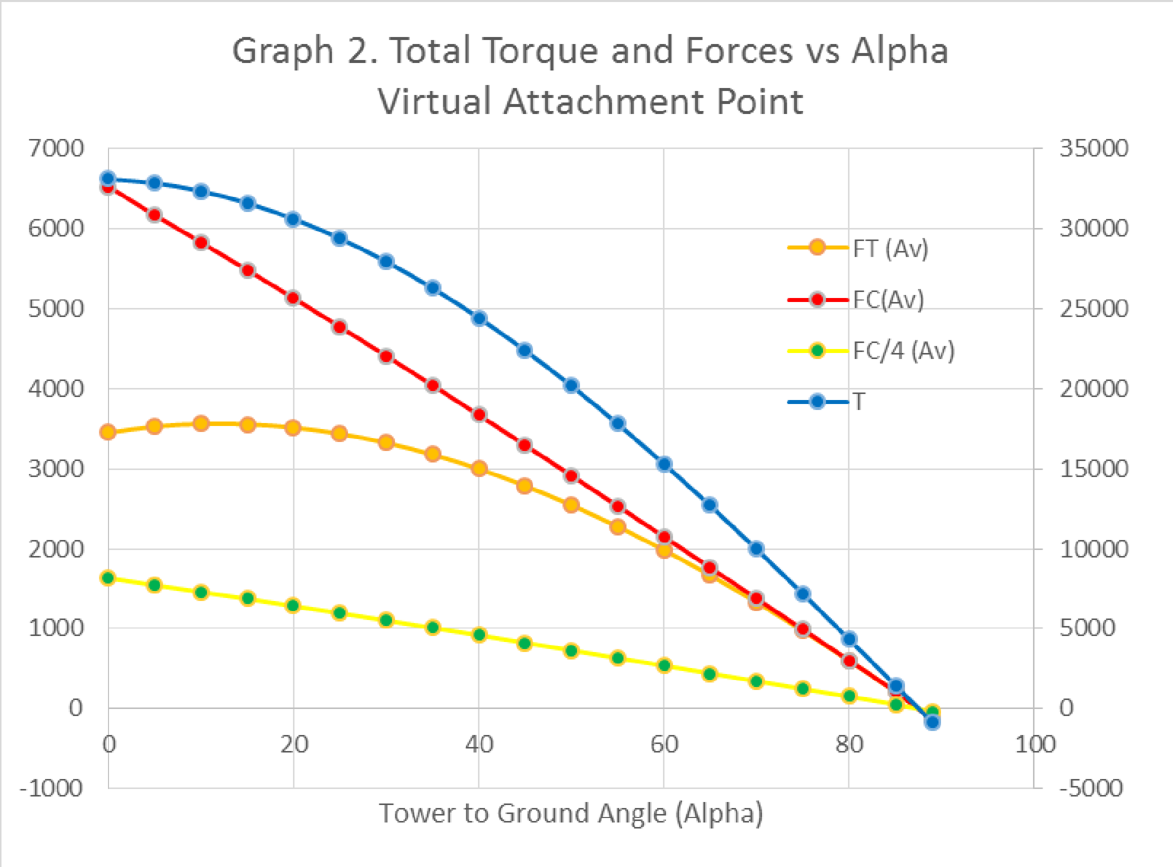
I had originally assumed that the counterclockwise torque calculations would have to be made assuming along the same axis as the center of mass for the loads. Thus I created the Virtual Attachment Point. After going through that exercise, I realized that this might not be the case and that any attachment point would give the same result of the angles and forces were calculated properly. For that reason an analysis was done using the Actual Attachment Point. A plot of the force on the winch, Fw (Also Fc/4), calculated from the Actual Attachment Point, as a function of tower angle, α, is indistinguishable to a plot of Fw calculated from the Virtual Attachment Point. The data is included in Chart 1 and Graph 4.

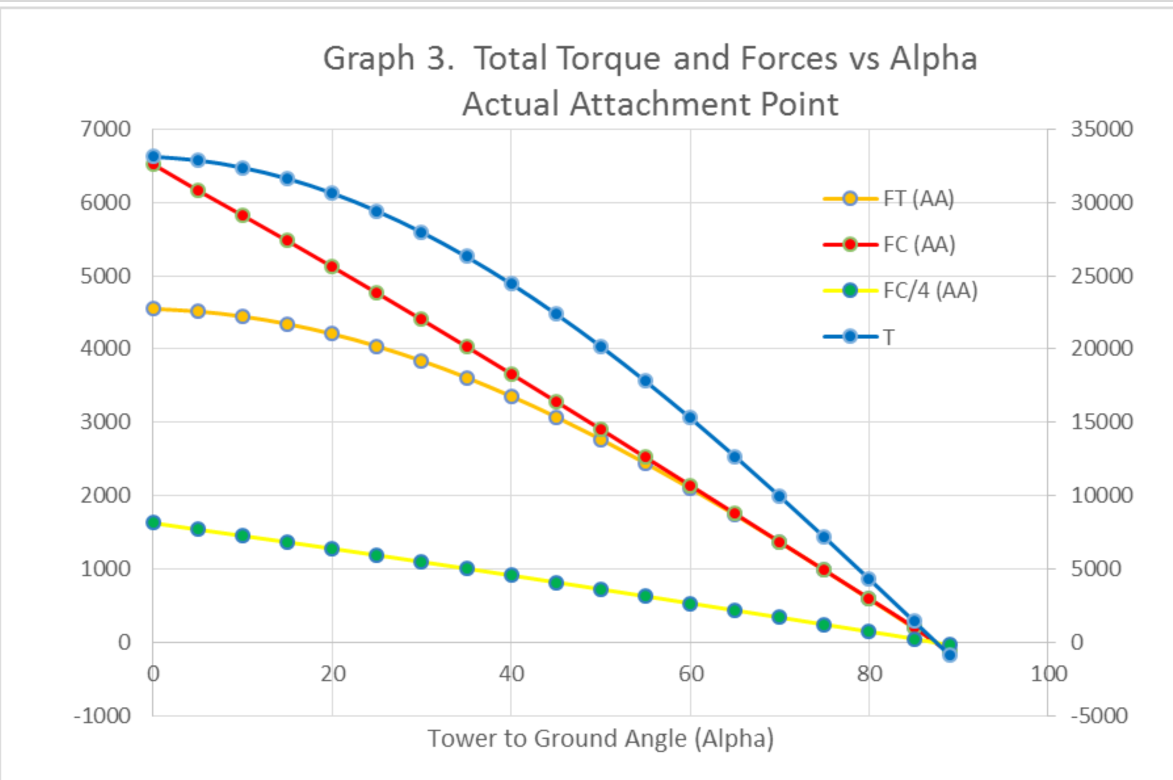
There was one anomalous observation. Theta, θ, and Psi, ψ, increase unexpectedly as Alpha, α, approaches 90°. See Graph 1. Inspection of the data reveals that the measurements for the calculation of Theta, θ, namely Lx and Ly, become very small as α approaches 90°. It is not clear to me if small errors in the calculation, an error in the model or if the actual angle that is being formed in this model is causing this apparent discrepancy or whether this is real phenomenon and the model is actually accurate. In any event, the numbers are small and any error in the number would be of no consequence during the operation of a winch or hoist.

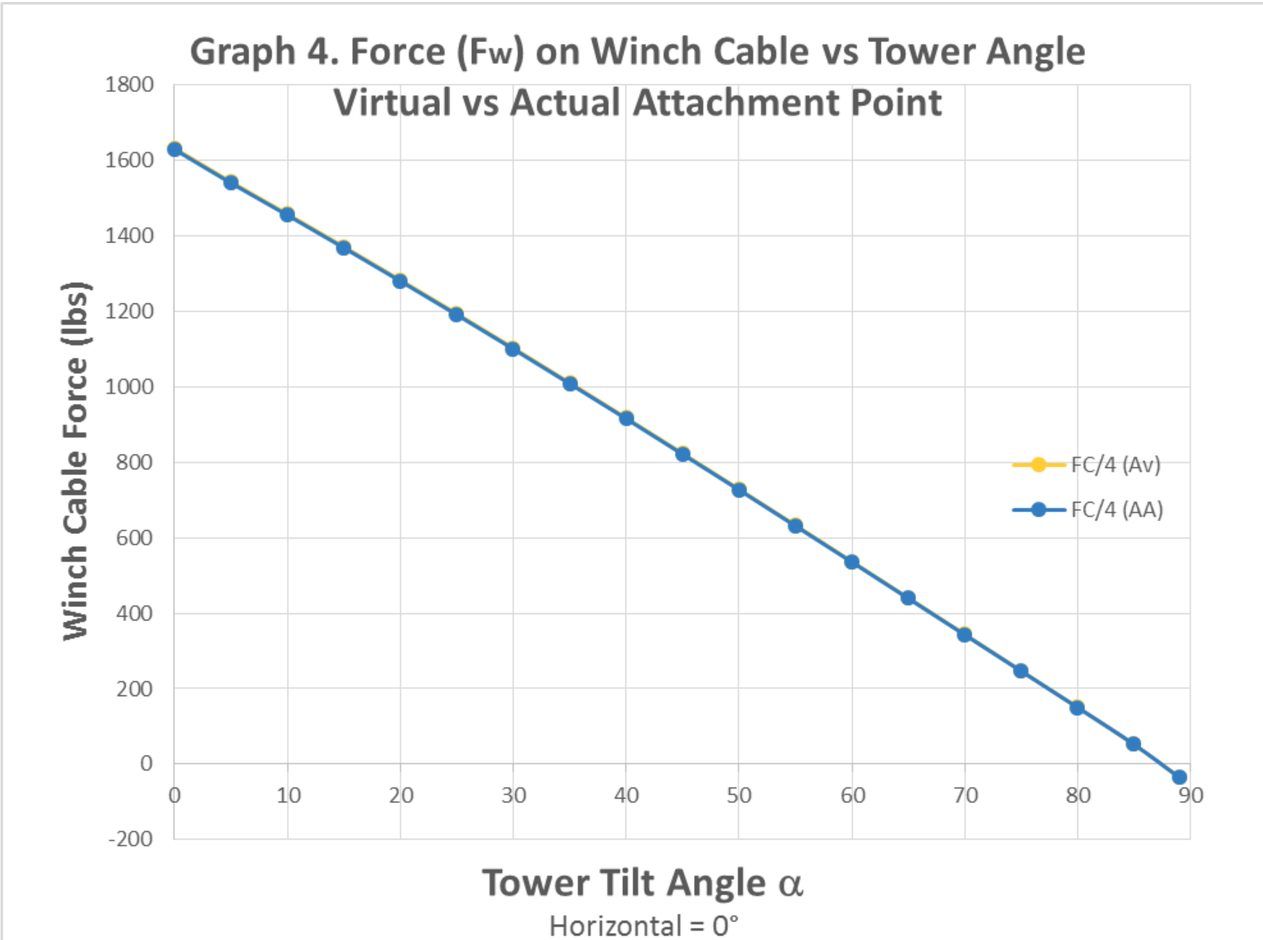
Table 1. Excel Spreadsheet Model V4.

Alpha	Sin α	Cos α	Ly	Lx	Ly/Lx	Arctan(Ly/Lx) (Psi)	Theta (θ)	COT θ	SIN θ	Av	∠Av	SIN (∠Av+θ)	T	F_T (Av)	F_C(Av)	F_C/4 (Av)	F_T (A_A)	F_C (A_A)	F_C/4 (A_A)
0	0.000	1.000	5.26	9.76	0.54	28.3	28.3	1.86	0.47	9.60	3.58	0.53	33116	3450	6530	1633	4549	6518	1629
5	0.087	0.996	4.70	9.56	0.49	26.2	31.2	1.65	0.52	9.32	3.69	0.57	32864	3528	6174	1544	4514	6162	1540
10	0.174	0.985	4.11	9.31	0.44	23.8	33.8	1.49	0.56	9.09	3.78	0.61	32363	3561	5834	1458	4445	5821	1455
15	0.259	0.966	3.56	9.01	0.39	21.5	36.5	1.35	0.60	8.89	3.86	0.65	31615	3557	5487	1372	4343	5474	1369
20	0.342	0.940	3.03	8.66	0.35	19.3	39.3	1.22	0.63	8.71	3.94	0.68	30627	3515	5134	1284	4207	5122	1281
25	0.423	0.906	2.53	8.26	0.31	17.0	42.0	1.11	0.67	8.55	4.01	0.72	29405	3438	4776	1194	4039	4765	1191
30	0.500	0.866	2.07	7.83	0.26	14.8	44.8	1.01	0.70	8.41	4.08	0.75	27960	3325	4413	1103	3841	4402	1100
35	0.574	0.819	1.65	7.35	0.22	12.6	47.6	0.91	0.74	8.28	4.15	0.79	26302	3178	4045	1011	3613	4034	1009
40	0.643	0.766	1.27	6.84	0.19	10.5	50.5	0.82	0.77	8.15	4.21	0.82	24444	2998	3673	918	3358	3663	916
45	0.707	0.707	0.94	6.30	0.15	8.5	53.5	0.74	0.80	8.04	4.27	0.85	22399	2787	3296	824	3077	3287	822
50	0.766	0.643	0.65	5.73	0.11	6.5	56.5	0.66	0.83	7.93	4.33	0.87	20185	2546	2916	729	2773	2908	727
55	0.819	0.574	0.42	5.14	0.08	4.7	59.7	0.59	0.86	7.82	4.39	0.90	17816	2278	2534	634	2447	2527	632
60	0.866	0.500	0.24	4.53	0.05	3.0	63.0	0.51	0.89	7.71	4.45	0.92	15312	1985	2150	537	2103	2143	536
65	0.906	0.423	0.11	3.91	0.03	1.6	66.6	0.43	0.92	7.61	4.51	0.95	12692	1669	1764	441	1743	1758	440
70	0.940	0.342	0.04	3.28	0.01	0.7	70.7	0.35	0.94	7.49	4.58	0.97	9975	1332	1377	344	1370	1373	343
75	0.966	0.259	0.02	2.64	0.01	0.5	75.5	0.26	0.97	7.36	4.66	0.99	7182	975	990	248	986	987	247
80	0.985	0.174	0.06	2.01	0.03	1.7	81.7	0.15	0.99	7.20	4.76	1.00	4334	602	603	151	595	601	150
85	0.996	0.087	0.15	1.38	0.11	6.3	91.3	-0.02	1.00	6.97	4.92	0.99	1453	209	210	52	200	209	52
89	1.000	0.017	0.27	0.89	0.30	16.7	105.7	-0.28	0.96	6.61	5.19	0.93	-860	-130	-139	-35	-118	-139	-35

			Torque Component	Force	Force Weight (lb)	C.O.M. Distance from Pivot		Component Angle	Cos (Angle)
A =	7	Horizontal Distance Pivot to Attachment	Tower	F1 =	2000	B =	11.5	2.99	0.998642
B =	11.5	Horizontal Distance to Tower COM	Rotor	F2 =	50	C =	18	1.91	0.999445
C =	18	Horizontal Distance to Rotor COM	Antenna	F3 =	250	D =	25	1.37	0.999712
D =	25	Horizontal Distance to Antenna COM	Mast	F4 =	100	E =	30	1.15	0.9998
E =	30	Horizontal Distance to Mast COM							
H =	2	Distance from Pivot to Common Reference Point							
G =	7.3	Length of Lever Post							
Gx =	0.76	Horizontal Distance of Lift Pulley from Common Reference Point = (G*Sinβ)							
Gy =	7.26	Vertical Height of Pulley							
J =	2	Tower Side Length							
r =	0.6	Distance from Tower Side to Tower Centroid							
Psi	ψ	Angle between the cable and horizontal							
Theta	θ	Angle between the tower and the cable. θ = α + ψ							
Alpha	α	Angle between the tower and horizontal							
P	2.763058	Horizontal Distance of Pulley from Pivot = H+(G*Sinβ)							
β	6	Angle of Retreat of Pulley Post							
COS β	0.995								
SIN β	0.105								
∠AA	15.9								
AA	7.3								


Graph 2. Total Torque and Forces vs Alpha
Virtual Attachment Point
FT (Av)
FC(Av)
FC/4 (Av)
T
Tower to Ground Angle (Alpha)
7000
6000
5000
4000
3000
2000
1000
0
-1000
35000
30000
25000
20000
15000
10000
5000
0
-5000
0
20
40
60
80
100


Graph 3. Total Torque and Forces vs Alpha
Actual Attachment Point
FT (AA)
FC (AA)
FC/4 (AA)
T
Tower to Ground Angle (Alpha)
7000
6000
5000
4000
3000
2000
1000
0
-1000
35000
30000
25000
20000
15000
10000
5000
0
-5000
0
20
40
60
80
100


Graph 4. Force (Fw) on Winch Cable vs Tower Angle
Virtual vs Actual Attachment Point
Winch Cable Force (lbs)
1800
1600
1400
1200
1000
800
600
400
200
0
-200
0
10
20
30
40
50
60
70
80
90
FC/4 (Av)
FC/4 (AA)
Tower Tilt Angle α
Horizontal = 0°